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The Surface Singularity or Boundary Integral Method Applied to Supercavitating Hydrofoils

James S. Uhlman, Jr.[1]

The surface singularity or boundary integral method is formulated numerically for the problem of the fully nonlinear potential flow past a supercavitating flat-plate hydrofoil. An iterative scheme is employed to locate the cavity surface. Upon convergence, the exact boundary conditions are satisfied on the foil-cavity boundary. The predictions of the nonlinear model are compared with those generated by linear theory and with experimental data. In contrast to the results for the partialy cavitating case, the predictions of the linear theory for supercavitating flat-plate hydrofoils are seen to be excellent.

Introduction

IN A RECENT PAPER, the author discussed the application of the surface singularity or boundary integral method to partially cavitating two-dimensional hydrofoils [1,2].[2] In the present paper, the case of supercavitating hydrofoils is treated with the same method.

The study of supercavitation may be said to have begun with the development of the hodograph method by Helmholtz and Kirchoff in the nineteenth century. Their method was extended to curved bodies by Levi-Civita and to finite cavitation numbers by Efros [3] and others (see Gilbarg and Serrin [4] and Shiffman [5,6]). All of these methods were nonlinear in the sense that the exact kinematic and dynamic potential-flow boundary conditions were satisfied on the cavity boundary. These methods also required a cavity termination model. The models most often used were the Riabouchinsky flat-plate cavity termination and the reentrant jet cavity termination.

Linearized theories were developed starting with Tulin [7] in response to the mathematical intractability of the exact theories. The linearized theories were extended by Tulin [8], Geurst [9], and Hanaoka [10], among others. More recently, numerical implementations of the linear model have found favor and extensions to three-dimensional unsteady flows have been developed (see Jiang [11] and Lee [12]).

These numerical extensions have, however, raised questions concerning the accuracy of the linearized theory. Of particular interest is the ability to predict unsteady cavity volume velocities and accelerations. In order for one to predict these quantities, the cavity volume predictions must be quite accurate.

The present paper presents an exact nonlinear numerical potential-flow model of the supercavitating flow about a two-dimensional hydrofoil. The predictions of this exact model are compared with those of the linear theory in order to ascertain the accuracy of the linear approximation. In contrast to the partially cavitating case [1,2], it is found that the agreement is excellent. The results of both theories are also compared with the experimental data of Wade and Acosta [13]. It is found that the agreement between theory and experiment, in general, is quite good.

[1]Naval Underwater Systems Center, Newport, Rhode Island.

[2]Numbers in brackets designate References at end of paper.

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Mathematical formulation

Following Uhlman [1], consider the unbounded, steady, irrotational flow of an inviscid, incompressible liquid past a cavitating hydrofoil. The flow is then a potential flow and hence possesses a potential function, Φ, which in the fluid satisfies Laplace's equation

$$\nabla^2\Phi = 0 \tag{1}$$

A disturbance potential, ϕ, can be defined by

$$\Phi = \mathbf{U}_\infty \cdot \mathbf{r} + \phi \tag{2}$$

where $\mathbf{U}_\infty$ is the freestream velocity vector and $\mathbf{r}$ is the position vector, $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$. All quantities may be made dimensionless with respect to ρ, $|\mathbf{U}_\infty|$ and c, where c is the chord of the hydrofoil. Unless otherwise stated, it shall henceforth be assumed that all terms are dimensionless. Equations (1) and (2) still hold, yet now in dimensionless form.

It is shown in Uhlman [1] that the disturbance potential, ϕ, and the disturbance velocity

$$\mathbf{V} = \nabla\phi \tag{3}$$

may be expressed in terms of an integral around the foil-cavity boundary which is linear in the unknown surface vorticity, γ. A similar formulation is employed here except that in the present paper

$$\mathbf{U}_\infty = 1\mathbf{i} \tag{4}$$

and any angle of attack of the foil is expressed in the body shape (see Fig. 1).

Boundary conditions

Following reference [1], the kinematic condition on the foil-cavity boundary, C, is

$$\frac{\partial\Phi}{\partial n} = 0 \quad \text{on} \quad C \tag{5}$$

or

$$\mathbf{n}\cdot\mathbf{V} = -\mathbf{n}\cdot\mathbf{U}_\infty \quad \text{on} \quad C \tag{6}$$

where $\mathbf{n}$ is the outward unit normal vector to the boundary. It should be noted that since the disturbance velocity, $\mathbf{V}$, is a linear function of the unknown surface vorticity, γ, this kinematic boundary condition is also linear in γ.

Similarly, a dynamic boundary condition must be applied

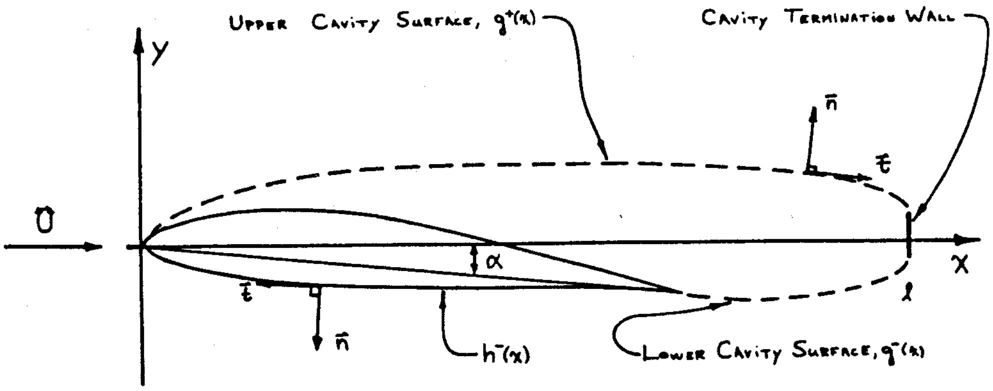

Fig. 1 Representative illustration of the supercavitating hydrofoil problem

on the cavity boundary. Using Bernoulli's equation, it may be shown that this condition becomes

$$Q^2 = 1 + \sigma \tag{7}$$

where σ is the cavitation number and

$$Q = |\mathbf{U}_\infty + \mathbf{V}| \tag{8}$$

This dynamic boundary condition may be reexpressed as

$$Q - \sqrt{1+\sigma} = 0 \tag{9}$$

in order to yield an expression which is linear in the unknowns γ and $\sqrt{(1+\sigma)}$. The solution procedure then requires that the cavity length be given and proceeds to find the cavity shape and the values of the above unknowns. For a given hydrofoil, this approach then yields the cavitation number as a single-valued function of the cavity length.

A detachment condition of some form is required at the trailing edge of the foil, in place of a Kutta condition, in order to set the circulation. In the general case, a leading-edge detachment condition would also be necessary. However, such a condition is beyond the scope of potential-flow theory and may be replaced, in the case of a flat-plate hydrofoil, by assuming that the upper cavity surface springs from the leading edge. The trailing-edge detachment condition for the supercavitating case is significantly different from the Kutta condition employed in the partially cavitating case. This is due to the fact that in a supercavitating case, only the lower surface of the hydrofoil trailing edge is wetted. The appropriate condition in this case is that the boundary streamline have a continuous first derivative. This condition is necessary if the dynamic boundary condition is to be satisfied everywhere on the streamline bounding the cavity. Equivalently, one could require that the surface velocity be continuous at the trailing edge of the hydrofoil. This condition may be expressed as

$$\lim_{A \to \text{T.E.}} \mathbf{t}_A \cdot (\mathbf{U}_\infty + \mathbf{V}_A) = \lim_{B \to \text{T.E.}} \mathbf{t}_B \cdot (\mathbf{U}_\infty + \mathbf{V}_B) \tag{10}$$

where A and B represent points on the boundary streamline on opposite sides of the trailing edge (see Fig. 2).

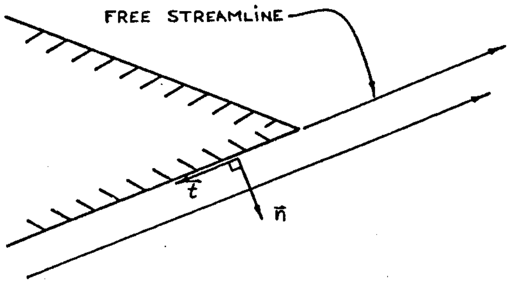

Fig. 2 Local trailing-edge flow for a supercavitating hydrofoil

A cavity termination model is also necessary, in this nonlinear model. In a manner quite similar to that employed in reference [1], a Riabouchinsky cavity termination model is adopted. The cavity termination wall for the supercavitating case is situated perpendicular to the free stream. As in the partially cavitating case, its height is initially unknown and a kinematic boundary condition is satisfied on its surface. The height is determined as part of the solution and is exactly that height required to connect the ends of the free streamlines springing from the leading and trailing edges of the hydrofoil (see Fig. 1).

The boundary conditions (6) and (9) are exact. Together with the trailing-edge detachment condition, the cavity termination model, and the given cavity length, they yield sufficient information to determine the unknown surface vorticity distribution, cavitation number, and cavity shape. There exist, however, alternate formulations of these boundary conditions which are better suited to a numerical solution. These alternate formulations of the boundary conditions are discussed in references [1] and [2]. The formulations employed in references [1] and [2] for the partially cavitating case are employed similarly for the supercavitating case presently under consideration. The solution procedure then consists of applying the kinematic boundary condition, (6), on the wetted portions of the foil boundary and the dynamic boundary condition, (9), on the assumed cavity boundary. The

Nomenclature

ρ = fluid density
c = hydrofoil chord length
$\mathbf{U}_\infty$ = freestream velocity
$\mathbf{V}$ = disturbance velocity
Q = total velocity magnitude
$\mathbf{n}$ = normal vector into fluid
$\mathbf{t}$ = tangent vector
$\mathbf{r}$ = position vector
α = angle of attack
σ = cavitation number, $2(p - p_c)/\rho U^2$
ℓ/c = cavity length to chord length ratio
C_L = lift coefficient, $2L/\rho U^2 c$
C_M = moment coefficient, $2M/\rho U^2 c^2$
Vol = dimensionless cavity volume, Volume/c^2
Φ = total velocity potential
ϕ = disturbance velocity potential
γ = surface vorticity distribution

boundary-value problem is then solved and the unknowns, γ and $\sqrt{(1 + \sigma)}$ are obtained. The kinematic boundary condition over the cavity boundary is then employed in order to obtain a better estimate of the cavity shape. This procedure is then repeated until the cavity shape has converged. See Uhlman [1] for a more thorough discussion.

Other quantities of interest

From the solution to the above boundary-value problem, other quantities of interest may be derived. In view of the fact that these derivations may be found in references [1] and [2], they shall not be reproduced here. Instead, we shall merely enumerate the quantities which subsequently will be presented: cavity volume (Vol), lift coefficient (C_L), moment coefficient (C_M). In addition to these, the quantities cavity length (ℓ), angle of attack (α), and cavitation number (σ), which are fundamental to the solution, shall be of most interest.

Results

The present numerical formulation is exact in the sense that as the number of surface elements tends to infinity and the size of the largest element tends to zero, the solution should converge to that of the given potential-flow problem. As discussed in reference [1], tests have been performed in order to determine the number of elements and iterations necessary to achieve convergence. The results shown in reference [1] are typical of the supercavitating case as well. Most of the data presented herein were calculated using 200 surface elements and 15 iterations. The only exceptions to this occur at long cavity lengths where up to 25 iterations were required.

Calculations in the supercavitating regime were performed only for the case of a flat plate hydrofoil. Figure 3 shows the final converged cavity shape for a supercavitating flat plate at 4-deg angle of attack and a cavity length of 1.4 times the chord length. The modified Riabouchinsky cavity termination model is apparent at the cavity trailing edge. Figure 4 shows the dimensionless pressure distribution associated with the cavity shape shown in Fig. 3. As expected, the stagnation points at the leading edge of the flat plate and at the trailing edge of the cavity are evident. There is no stagnation point at the trailing edge of the hydrofoil, in accordance with the trailing-edge detachment condition discussed above.

Figure 5 shows the dimensionless cavity length, ℓ/c, as a function of α/σ for a supercavitating flat-plate hydrofoil. Included are the present nonlinear results, the predictions of the linear theory of Geurst [9] and the experimental data of Wade and Acosta [13]. It is evident that the theoretical predictions are in excellent agreement with one another and are consistent with the experimental data.

Figures 6 and 7 show the corresponding results for the lift and moment coefficients, respectively. Both quantities have been normalized with respect to their linear theory zero cavitation number limits. The moment has been taken about the midchord of the hydrofoil. The agreement between the two theoretical approaches is good. The agreement of the theoretical predictions with the experimental data of Wade and Acosta [13] for the lift coefficient is also quite good. The agreement of the predicted moment coefficient with the experimental data is, however, unsatisfactory. The reasons for this disagreement are not clear.

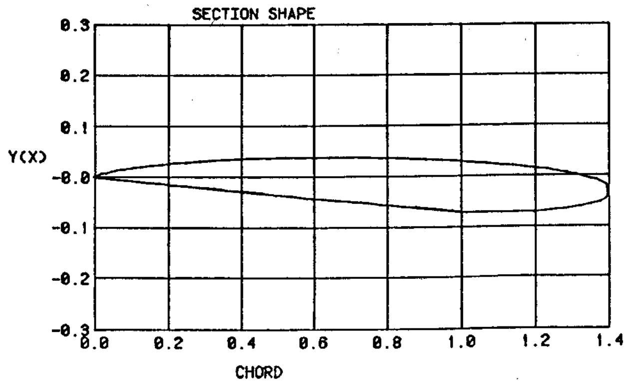

Fig. 3 Converged cavity shape for a supercavitating flat-plate hydrofoil at 4-deg angle of attack with a cavity length of 1.40 after 25 iterations

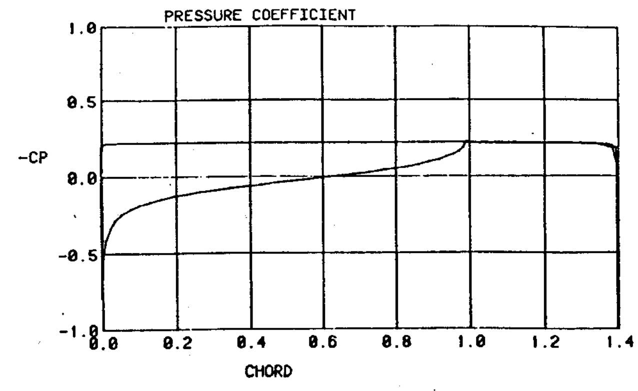

Fig. 4 Pressure distribution over a supercavitating flat-plate hydrofoil at 4-deg angle of attack with a cavity length of 1.40 after 25 iterations

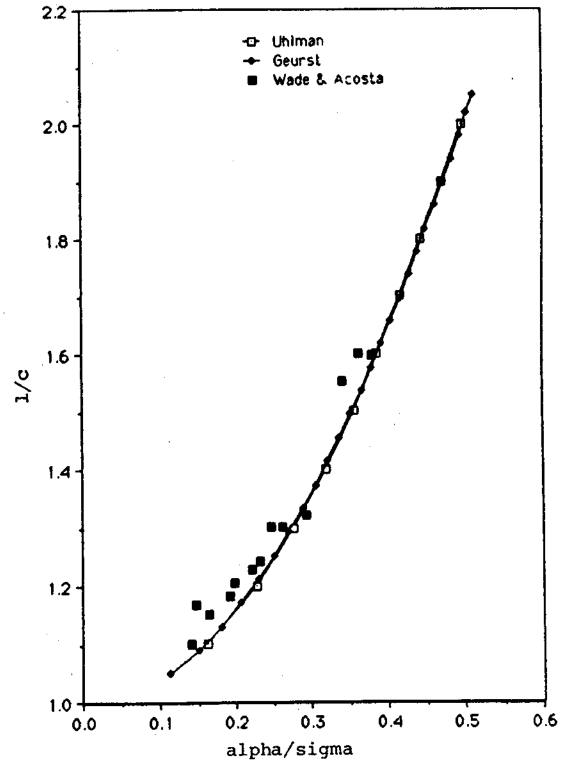

Fig. 5 Cavity length, ℓ/c, versus α/σ for a supercavitating flat-plate hydrofoil. The theoretical predictions of Uhlman [1] and Geurst [9], and the experimental data of Wade and Acosta [13] are presented

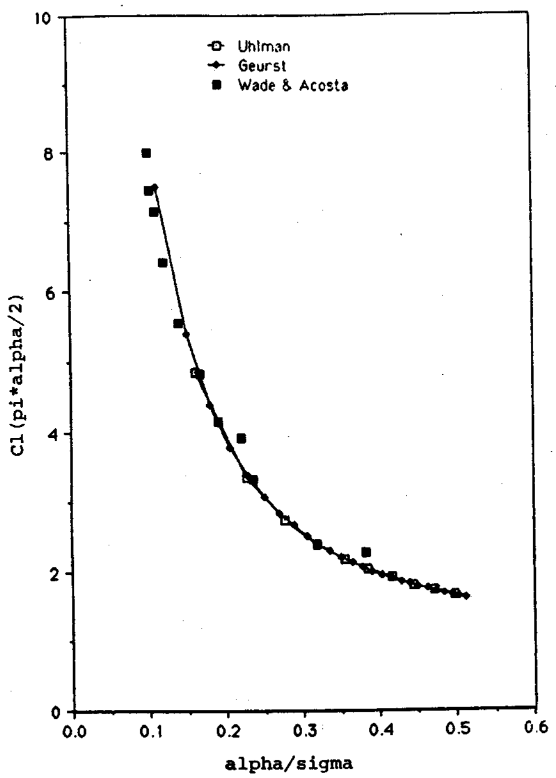

Fig. 6 Normalized lift coefficient, $C_L/(\pi\alpha/2)$, versus α/σ for a supercavitating flat-plate hydrofoil. The theoretical predictions of Uhlman [1] and Geurst [9], and the experimental data of Wade and Acosta [13] are presented

Finally, a comparison between the theoretical predictions for the cavity volume is presented in Fig. 8. Again the agreement is seen to be excellent for sufficiently large α/σ or, equivalently, for sufficiently large ℓ/c. It should be noted that the linear theory predicts that the cavity volume becomes unbounded as the length of the cavity approaches one hydrofoil chord length from above.

Conclusions

As is evident from the figures presented herein, the predictions of the two theoretical approaches are in good agreement for values of the parameter α/σ exceeding roughly 0.3, corresponding to a cavity to chord length ratio, ℓ/c, of approximately 1.30. For values of this parameter less than 0.3 the moment coefficient, C_M, and cavity volume, Vol, predicted by the linear and nonlinear theories diverge, although the cavity extent, ℓ/c, and lift coefficient, C_L, are still in good agreement.

The agreement of both theories with the experimental data is, in general, satisfactory. The predicted values of both the cavity extent, ℓ/c, and the lift coefficient, C_L, are readily seen to be in excellent agreement with the data of Wade and Acosta [13] over the range examined here (see Figs. 5 and 6). The values predicted by the two theories for the moment coefficient about the midchord of the hydrofoil, C_M, are, however, not in good agreement with the corresponding experimental data (Fig. 7). The cause for this disagreement is unknown, although the general agreement between the two theories and the experimental data otherwise suggest that the problem may lie elsewhere than with the theory. No experimental data have been plotted for comparison with the theoretical predictions of the cavity volume due to the fact that no such data exist.

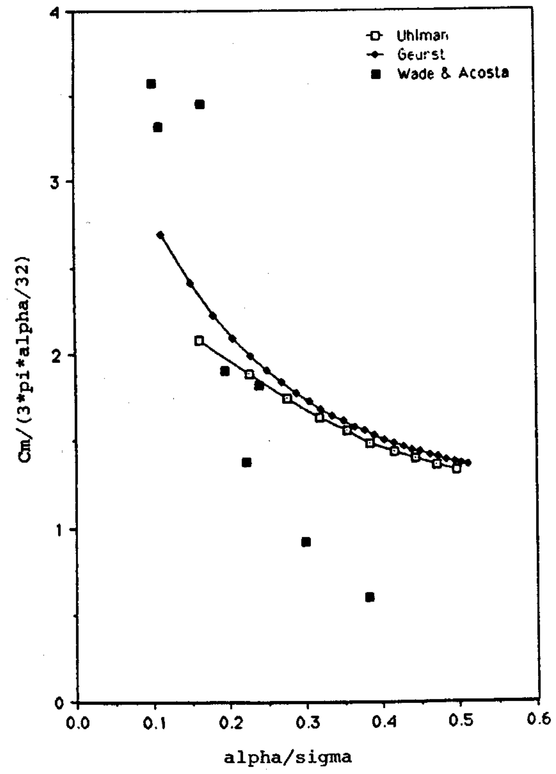

Fig. 7 Normalized moment coefficient, $C_M/(3\pi\alpha/32)$, versus α/σ for a supercavitating flat-plate hydrofoil. The theoretical predictions of Uhlman [1] and Geurst [9], and the experimental data of Wade and Acosta [13] are presented

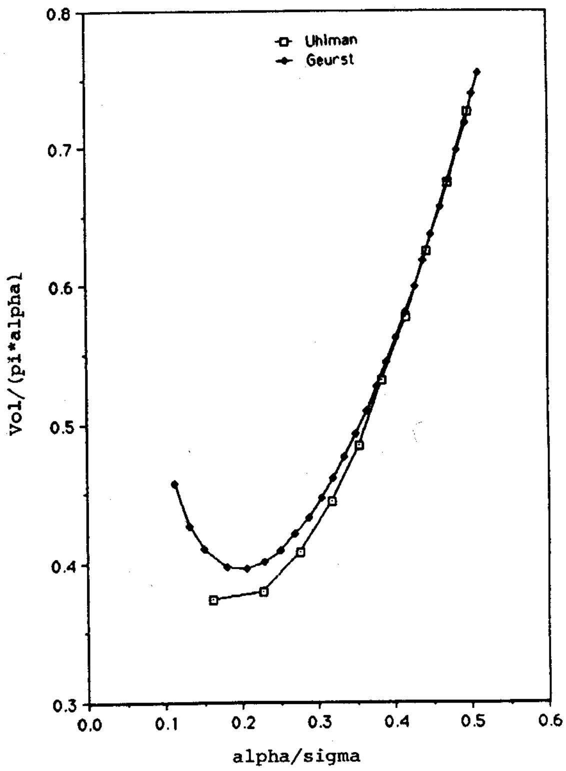

Fig. 8 Normalized dimensionless cavity volume, $Vol/\pi\alpha$, versus α/σ for a supercavitating flat-plate hydrofoil. The theoretical predictions of Uhlman [1] and Geurst [9] are presented

As shown in Fig. 8, the predictions of the linear theory and the nonlinear theory are seen to be in substantive agreement for α/σ greater than about 0.3 or ℓ/c greater than about 1.30. The disagreement of the cavity volume predictions for short cavity lengths is to be expected, since the linear theory does not even predict a finite value for this quantity when ℓ/c equals 1.00.

In general, it appears that the linear theory of Geurst [9] for supercavitating flat plate hydrofoils is in excellent agreement with the nonlinear theory of Uhlman [1]. The only exception to this conclusion occurs at short cavity lengths. This result is in sharp contrast to that obtained for partially cavitating hydrofoils (see Uhlman [1,2]) wherein the predictions of the linear theory were suspect. The agreement of both theories with the experimental data of Wade and Acosta [13] is also quite good with the exception of the moment coefficient. Thus it may be concluded that the linear theory for supercavitating flat-plate hydrofoils may be employed with confidence in the prediction of forces, cavity extents, and volumes.

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References

1 Uhlman, J. S., "The Surface Singularity Method Applied to Partially-cavitating Hydrofoils," Ph.D. thesis, Department of Ocean Engineering, M.I.T., Cambridge, Mass., 1983.

2 Uhlman, J. S., "The Surface Singularity Method Applied to Partially-Cavitating Hydrofoils," Journal of Ship Research, Vol. 31, No. 2, June 1987.

3 Efros, D. A., "Hydrodynamic Theory of Two-Dimensional Flow with Cavitation," *Doklady Akademii Nauk. SSSR*, Vol. 51, 1946, pp. 267–270.

4 Gilbarg, D. and Serrin, J., "Free Boundaries and Jets in the Theory of Cavitation." *Journal of Mathematics and Physics*, Vol. 29, 1950, pp. 1–12.

5 Shiffman, M., "On Free Boundaries of an Ideal Fluid—I," *Communications on Pure and Applied Mathematics*, 1948, pp. 89–99.

6 Shiffman, M., "On Free Boundaries of an Ideal Fluid—II," *Communications on Pure and Applied Mathematics*, 1949, pp. 1–11.

7 Tulin, M. P., "Steady Two-Dimensional Cavity Flows about Slender Bodies," DTMB Report 834, David Taylor Model Basin, Washington, DC, 1953.

8 Tulin, M. P., "Supercavitating Flows—Small Perturbation Theory," Journal of Ship Research, Vol. 7, No. 3, Jan. 1964.

9 Geurst, J. A., "Linearized Theory for Fully Cavitating Hydrofoils," *International Shipbuilding Progress*, Vol. 7, No. 65, 1960.

10 Hanaoka, T., "Linearized Theory of Cavity Flow Past a Hydrofoil of Arbitrary Shape," *Journal of the Society of Naval Architects of Japan*, Vol. 115, 1964.

11 Jiang, C. W., "Experimental and Theoretical Investigations of Unsteady Super-cavitating Hydrofoils of Finite Span," Ph.D. thesis, Department of Ocean Engineering, M.I.T., Cambridge, Mass., 1977.

12 Lee, C. S., "Prediction of Steady and Unsteady Performance of Marine Propellers with or without Cavitation by Numerical Lifting Surface Theory," Ph.D. thesis, Department of Ocean Engineering, M.I.T., Cambridge, Mass., 1979.

13 Wade, R. B. and Acosta, A., "Experimental Observations on the Flow Past a Plano-Convex Hydrofoil," *Journal of Basic Engineering*, March 1966.