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An Examination of the Frequencies of the Unsteady Harmonic Forces Generated by Propulsors

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PREFACE

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13. ABSTRACT ***(Maximum 200 words)***

An analysis is presented of the expected frequencies generated by the interaction between the blade rows of multiple blade row propulsors and by the interactions of the blade rows with a circumferentially nonuniform inflow field. These results are then employed in the determination of the frequencies expected to be found in a variety of propulsor and control surface configurations. The implications of these findings for the silencing of propulsor tonal noise are then discussed. This study assumes that the blades on all blade rows are uniformly spaced.

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LIST OF ABBREVIATIONS AND SYMBOLS

M	Number of blades in the forward blade row
N	Number of blades in the aft blade row
m	Shaft rotation rate of the forward blade row
n	Shaft rotation rate of the aft blade row
θ_i	Angular position of the i-th blade of the forward blade row
φ_j	Angular position of the j-th blade of the aft blade row
T	Thrust force
H	Horizontal force

AN EXAMINATION OF THE FREQUENCIES OF THE UNSTEADY HARMONIC FORCES GENERATED BY PROPULSORS

INTRODUCTION

Among the noise components of concern in weapon propulsors are the tonals generated by the interaction of the blade rows with the periodic, circumferentially and radially varying inflow field experienced by the propulsor and by the interaction of the blade rows with each other. The frequencies at which these tonals occur depend on the periodicity of the inflow, the rotation rates of the blade rows, and the number of blades in each blade row. These interactions have been investigated previously by Strasberg and Breslin[1] and Thompson.[2] This report may be considered to be an extension of these efforts, which applies the results derived therein to a variety of propulsor configurations. There do, however, exist nonlinear interactions that can produce tonals at frequencies other than the ones determined here. Fortunately, the magnitudes of these other tonals tend to be lower than the magnitudes of the tonals due to these linear interactions.

The interactions of a blade row with the periodic, nonaxisymmetric inflow and the interactions of a blade row with another blade row will be examined separately. The frequencies at which the unsteady thrust and side forces occur will be examined. The results of these analyses will then be employed in an examination of the frequencies expected in a variety of propulsor/control surface configurations typical of those employed as weapon propulsors.

MATHEMATICAL FORMULATION

UNSTEADY THRUST INTERACTIONS OF TWO BLADE ROWS

Suppose that the forward blade row has M identical, symmetrically spaced blades and rotates with an angular frequency of m revolutions per second (rotation rates are negative for right-handed blade rows and positive for left-handed blade rows). Suppose further that the aft blade row has N identical, symmetrically placed blades, and rotates with an angular frequency of n revolutions per second. The individual blades of the forward blade row shall be denoted by the index i, which goes from 0 to M-1 and the angle that the i-th blade makes with the upward vertical direction shall be denoted by θ_i. The individual blades of the aft blade row shall be denoted by the index j which goes from 0 to N-1 and the angle that the jth blade makes with the upward vertical direction shall be denoted by φ_j. These angles will vary with time as

$$\theta_i = 2\pi mt + 2\pi\left(\frac{i}{M}\right), \qquad (1)$$

and

$$\varphi_j = 2\pi nt + 2\pi\left(\frac{j}{N}\right) + \phi_0, \tag{2}$$

where the angle ϕ_0 denotes the angular displacement of the aft blade row relative to the first at $t = 0$. The coordinate system assumed in this investigation is presented in figure 1. The fluctuating thrust developed by the aft blade row will be considered first.

The fluctuating thrust T_j developed by the j-th blade of the aft blade row is assumed to be a periodic function of the angle Ω_j between that blade and the zero-th forward blade. Since there are M identical, symmetrically spaced forward blades the fluctuating force will possess a periodicity of $2\pi/M$ rather than simply 2π. The fluctuating thrust on the j-th blade is then expressible as a Fourier series as

$$T_j = \sum_{p=1}^{\infty} A_p \cos\left(pM\Omega_j - \alpha_p\right), \tag{3}$$

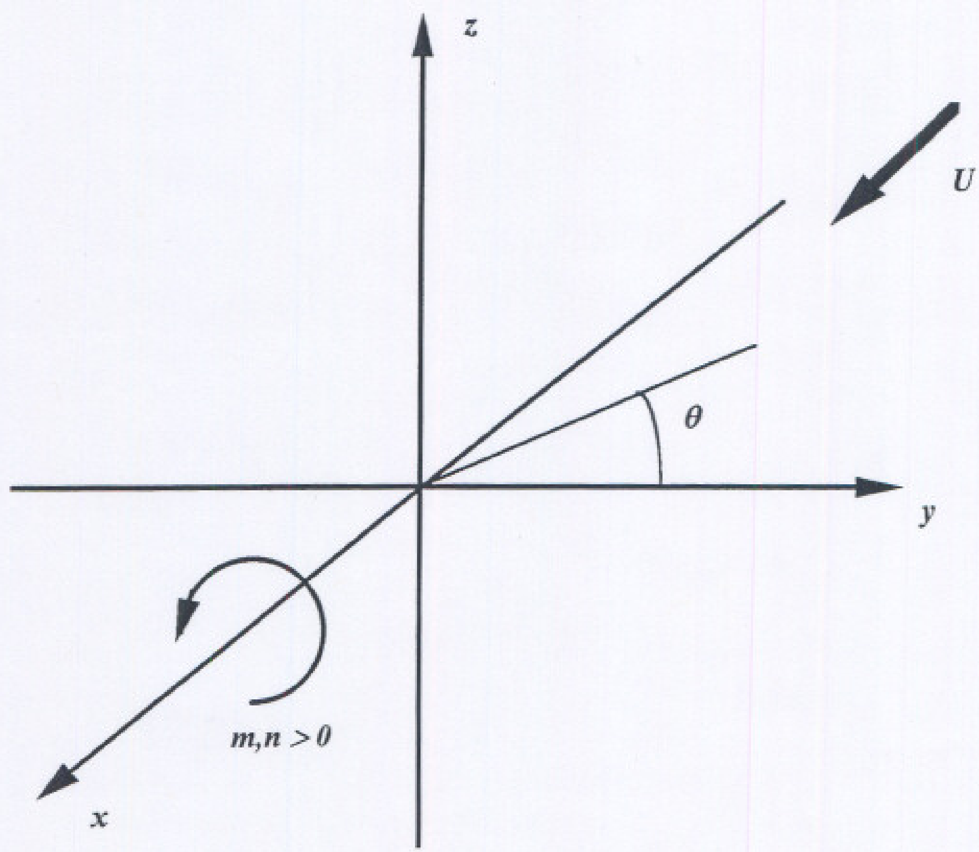

Figure 1. Coordinate System for the Propulsor

where A_p is the amplitude of the component, α_p is its phase and Ω_j is given by

$$\Omega_j = \varphi_j - \theta_0. \tag{4}$$

The total unsteady thrust on the aft blade row is then given by a simple summation over all the blades; that is

$$T = \sum_{j=0}^{N-1} \sum_{p=1}^{\infty} A_p \cos\left[pM\left(\varphi_j - \theta_0\right) - \alpha_p\right], \tag{5}$$

or

$$T = \sum_{j=0}^{N-1} \sum_{p=1}^{\infty} A_p \cos\left[pM\left(2\pi m t + 2\pi\left(\frac{j}{N}\right) + \phi_0 - \theta_0\right) - \alpha_p\right]. \tag{6}$$

By interchanging the orders of summation and employing the trigonometric identity (see Gradshteyn and Ryzhik[3]),

$$\sum_{r=0}^{R} \cos(a + rb) = \cos\left(a + \frac{1}{2}Rb\right) \frac{\sin\left[\frac{1}{2}(R+1)b\right]}{\sin\left[\frac{1}{2}b\right]}, \tag{7}$$

it can be found that

$$T = \sum_{p=1}^{\infty} A_p \cos\left[pM\left(2\pi m t + \phi_0 - \theta_0\right) - \alpha_p + \pi\frac{(N-1)}{N}pM\right] \frac{\sin[\pi p M]}{\sin\left[\pi\frac{pM}{N}\right]}. \tag{8}$$

Examination of the quotient of the two sines shows that the unsteady thrust contribution of the *k*-th harmonic will be zero unless there exists an integer q such that

$$pM = qN. \tag{9}$$

In this case, the sine quotient then becomes

$$\frac{\sin(\pi q N)}{\sin(\pi q)} = N(-1)^{q(N-1)}. \tag{10}$$

Combining these results and using the fact that

$$\theta_0 = 2\pi m t, \tag{11}$$

it can be found that the fluctuating thrust may be expressed as

$$T=\sum_{p=1}^{\infty}A_p\cos\left[2\pi(qNn-pMm)t-pM\phi_0-\alpha_p+\pi\frac{(N-1)}{N}pM\right]\left(\frac{\sin[\pi pM]}{\sin\left[\frac{pM}{N}\right]}\right). \qquad (12)$$

The associated frequencies will be given by

$$f_{pq}=qN|n|+pM|m|, \qquad (13)$$

for integers p and q such that

$$pM=qN. \qquad (14)$$

UNSTEADY SIDE FORCE INTERACTIONS OF TWO BLADE ROWS

In a similar manner, consider the unsteady side force interactions between two blade rows. The fluctuating tangential force S_j developed by the j-th aft blade is assumed to be a periodic function of the angle Ω_j between that blade and the zero-th forward blade. The fluctuating tangential force on the j-th blade is then expressible as a Fourier series as

$$S_j=\sum_{p=1}^{\infty}B_p\cos(pM\Omega_j-\beta_p), \qquad (15)$$

where B_p is the amplitude of the component and β_p is its phase. The horizontal component of this tangential force is given by

$$H_j=S_j\cos(\varphi_j), \qquad (16)$$

The total unsteady horizontal force is then given by a simple summation over all the blades; that is,

$$H=\sum_{j=0}^{N-1}\sum_{p=1}^{\infty}B_p\cos\left[pM(\varphi_j-\theta_0)-\beta_p\right]\cos(\varphi_j), \qquad (17)$$

or

$$H=\frac{1}{2}\sum_{j=0}^{N-1}\sum_{p=1}^{\infty}B_p\left\{\cos\left[(pM+1)\varphi_j-pM\theta_0-\beta_p\right]+\cos\left[(pM-1)\varphi_j-pM\theta_0-\beta_p\right]\right\}, \qquad (18)$$

using the trigonometric identity,

$$cos(a)\,cos(b) = \frac{1}{2}\left[cos(a+b) + cos(a-b)\right]. \tag{19}$$

By interchanging the orders of summation and employing the trigonometric identity given in equation (7), we find that

$$H = \frac{1}{2}\sum_{p=1}^{\infty} B\left\{ cos\left[(pM+1)(2\pi nt+\phi_0) - pM\theta_0 - \beta_p + \pi\frac{(pM+1)(N-1)}{N}\right]\left\{\frac{sin\left[\pi(pM+1)\right]}{sin\left[\pi\frac{(pM+1)}{N}\right]}\right\}\right.$$

$$\left. + cos\left[(pM-1)(2\pi nt+\phi_0) + pM\theta_0 - \beta_p + \pi\frac{(pM-1)(N-1)}{N}\right]\left\{\frac{sin\left[\pi(pM-1)\right]}{sin\left[\pi\frac{(pM-1)}{N}\right]}\right\} \right\}. \tag{20}$$

Examination of the quotients of the two sines shows that the unsteady thrust contribution of the *p*-th harmonic will be zero unless there exists an integer *q* such that either

$$pM + 1 = qN, \tag{21}$$

or

$$pM - 1 = qN. \tag{22}$$

The sine quotient then becomes

$$\frac{sin(\pi qn)}{sin(\pi q)} = N(-1)^{q(N-1)}, \tag{23}$$

the frequencies associated with these contributions will again be

$$f_{pq} = qN|n| + pM|m|, \tag{24}$$

where *p* and *q* must be integers such that

$$pM \pm 1 = qN. \tag{25}$$

UNSTEADY TORQUE AND MOMENT INTERACTIONS OF TWO BLADE ROWS

In a similar manner, consider the unsteady torque and moment interactions between two blade rows; it may be readily shown that the frequencies at which the unsteady torque harmonics occur are the same as those at which the unsteady thrust harmonics occur. Similarly, the frequencies at which the unsteady side moments arise and may be shown to be the same as those of the unsteady side forces.

RESULTS

INTERACTION BETWEEN TWO BLADE ROWS COUNTER-ROTATING AT EQUAL SPEEDS

In this case, let M be equal to the number of blades on the forward blade row and m be its rotation speed and N equal the number of blades on the aft blade row and $n = -m$ its rotation speed. Unsteady thrust interactions will then occur at frequencies for which there exist integers p and q such that

$$pM = qN, \tag{30}$$

and the frequencies associated with these interactions will be

$$f_{pq} = |n|(pM + qN). \tag{31}$$

Similarly, unsteady side force interactions will occur at frequencies for which there exist integers p and q such that

$$pM \pm 1 = qN, \tag{32}$$

and the frequencies associated with these interactions will again be at

$$f_{pq} = |n|(pM + qN). \tag{33}$$

These unsteady thrust interaction frequencies will be as shown in table 1. In this table, the first two thrust interaction frequencies are shown on the diagonal and below the diagonal, while the first two side force interaction frequencies are shown above the diagonal. Ellipses denote configurations at which the unsteady side force harmonics should have zero magnitude. Note that since both blade rows are rotating, the resultant frequencies depend on the sum of the blade row harmonics. Thus, these frequencies are not dependent on whether the forward blade row has more blades than the aft blade row or vice versa. No assumptions have been made regarding the magnitude of the blade force harmonics.

Table 1. Interaction Frequencies of Two Blade Rows Counterrotating at the Same Rate

	4	5	6	7	8	9	10	11	12	13
4	8/15	9/31	---	15/41	---	17/55	---	23/65	---	25/79
5	40/80	10/20	11/49	29/41	31/49	19/71	--	21/89	49/71	51/79
6	24/48	60/120	12/24	13/71	---	---	---	23/109	---	25/131
7	56/112	70/140	84/168	14/28	15/97	55/71	41/99	43/111	71/97	27/155
8	16/32	80/160	48/96	56/112	16/32	17/127	---	65/111	---	79/129
9	72/144	990/180	36/72	126/252	144/288	18/36	19/161	89/109	---	53/181
10	40/80	20/40	60/120	140/280	80/160	180/360	20/40	21/199	---	79/181
11	88/176	110/220	132/264	154/308	176/352	198/396	220/440	22/44	23/241	131/155
12	24/48	120/240	24/48	168/336	48/96	72/144	120/240	264/528	24/48	25/287
13	104/208	130/260	156/312	182/364	208/416	234/468	260/520	286/572	312/624	26/52

INTERACTION OF A ROTATING AFT BLADE ROW WITH A NONROTATING FORWARD BLADE ROW (PRESWIRL STATOR)

In this case, we may employ the preceding results with M equal to the number of blades on the nonrotating forward blade row and $m = 0$ its rotation speed and N equal to the number of blades on the rotating aft blade row and n its rotation speed. Unsteady thrust interactions will then occur at frequencies for which there exist integers p and q such that

$$pM = qN, \tag{26}$$

and the frequencies associated with these interactions will be

$$f_{pq} = qNn. \tag{27}$$

Similarly, unsteady side force interactions will occur at frequencies for which there exist integers p and q such that

$$pM \pm 1 = qN, \tag{28}$$

and the frequencies associated with these interactions will again be at

$$f_{pq} = qNn. \tag{29}$$

These unsteady thrust and side force interaction frequencies are shown in tables 2 and 3. Since only one blade row is rotating, the tonal frequencies resulting are dependent on the specific number of blades in the rotating blade row. Hence, table 2 presents the frequencies resulting when the forward blade row has more blades than the aft blade row, and table 3 presents the resulting frequencies when the aft blade row has more blades than the forward blade row. The configurations of these tables are similar to that of table 1.

Table 2. Interaction Frequencies of a Rotating Aft Blade Row with a Nonrotating Forward Blade Row ($M \geq N$).

	4	5	6	7	8	9	10	11	12	13
4	4/8	4/16	---	8/20	---	8/28	---	12/32	---	12/40
5	20/40	5/10	5/25	15/20	15/25	10/35	--	10/45	25/35	25/40
6	12/24	30/60	6/12	6/36	---	---	---	12/54	---	12/66
7	28/56	35/70	42/84	7/14	7/49	28/35	21/49	21/56	35/49	14/77
8	8/16	40/180	24/48	56/112	8/16	8/64	---	32/56	---	40/64
9	36/72	45/90	18/36	63/126	72/144	9/18	9/81	45/54	---	27/90
10	20/40	10/20	30/60	70/140	40/80	90/180	10/20	10/100	---	40/90
11	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	11/121	66/77
12	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	12/144
13	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26

Table 3. Interaction Frequencies of a Rotating Aft Blade Row with a Nonrotating Forward Blade Row ($M \leq N$).

	4	5	6	7	8	9	10	11	12	13
4	4/8	5/15	---	7/21	---	8/27	---	11/33	---	13/39
5	20/40	5/10	6/24	14/21	16/24	9/36	--	11/44	24/36	26/39
6	12/24	30/60	6/12	7/35	---	---	---	11/55	---	13/65
7	28/56	35/70	42/84	7/14	8/48	27/36	20/50	22/55	36/48	13/78
8	8/16	40/180	24/48	56/112	8/16	9/63	---	33/55	---	39/65
9	36/72	45/90	18/36	63/126	72/144	9/18	10/80	44/55	---	26/91
10	20/40	10/20	30/60	70/140	40/80	90/180	10/20	11/99	---	39/91
11	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	12/120	65/78
12	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	13/143
13	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26

INTERACTION OF A ROTATING FORWARD BLADE ROW WITH A NONROTATING AFT BLADE ROW (POSTSWIRL STATOR)

In this case, we may employ the preceding results with N equal to the number of blades on the nonrotating aft blade row and $n = 0$ its rotation speed and M equal to the number of blades on the rotating forward blade row and m its rotation speed. Unsteady thrust interactions will again occur at frequencies for which there exist integers p and q such that

$$pM = qN, \tag{30}$$

and the frequencies associated with these interactions will be

$$f_{pq} = pMm. \tag{31}$$

Similarly, unsteady side force interactions will occur at frequencies for which there exist integers p and q such that

$$pM \pm 1 = qN, \tag{32}$$

and the frequencies associated with these interactions will again be at

$$f_{pq} = pMm. \tag{33}$$

These unsteady thrust and side force interaction frequencies are shown in tables 4 and 5. Again, table 4 presents the frequencies resulting when the forward blade row has more blades than the aft blade row and table 5 presents the resulting frequencies when the aft blade row has more blades than the forward blade row. The configurations of these tables are again similar to that of table 1.

Table 4. Interaction Frequencies of a Rotating Forward Blade Row with a Nonrotating Aft Blade Row ($M \geq N$).

	4	5	6	7	8	9	10	11	12	13
4	4/8	5/15	---	7/21	---	9/27	---	11/33	---	13/39
5	20/40	5/10	6/24	14/21	16/24	9/36	--	11/44	24/36	26/39
6	12/24	30/60	6/12	7/35	---	---	---	11/55	---	13/65
7	28/56	35/70	42/84	7/14	8/48	27/36	20/50	22/55	36/48	13/78
8	8/16	40/180	24/48	56/112	8/16	9/63	---	33/55	---	39/65
9	36/72	45/90	18/36	63/126	72/144	9/18	10/80	44/55	---	26/91
10	20/40	10/20	30/60	70/140	40/80	90/180	10/20	11/99	---	39/91
11	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	12/120	65/78
12	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	13/143
13	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26

Table 5. Interaction Frequencies of a Rotating Forward Blade Row with a Nonrotating Aft Blade Row ($M \leq N$)

	4	5	6	7	8	9	10	11	12	13
4	4/8	4/16	---	8/20	---	8/28	---	12/32	---	12/40
5	20/40	5/10	5/25	15/20	15/25	10/35	--	10/45	25/35	25/40
6	12/24	30/60	6/12	6/36	---	---	---	12/54	---	12/66
7	28/56	35/70	42/84	7/14	7/49	28/35	21/49	21/56	35/49	14/77
8	8/16	40/180	24/48	56/112	8/16	8/64	---	32/56	---	40/64
9	36/72	45/90	18/36	63/126	72/144	9/18	9/81	45/54	---	27/90
10	20/40	10/20	30/60	70/140	40/80	90/180	10/20	10/100	---	40/90
11	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	11/121	66/77
12	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	12/144
13	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26

DISCUSSION

The tables presented in this report list the frequencies at which interactions between blade rows occur, but do not provide any information on the magnitude of the associated tonals. This frequency information is usually employed to select the number of blades in interacting blade rows under the assumption that the magnitude of the interaction tonals decrease with increasing harmonic number. With this assumption as a basis, the number of blades in each blade row are selected to provide the highest interaction frequencies possible. The strongest interaction frequencies are often those associated with the interactions of the rotating blade rows with upstream control surfaces or fins, which are usually four in number and uniformly spaced around the circumference. It may be readily seen from the above tables that reasonable selections for the blade number of rotating blade rows do not sufficiently raise the frequencies for this interaction. Possible approaches for countering this problem are: to put the control surfaces downstream of the propulsor (since the unsteady forces induced on an upstream blade row by a downstream blade row are small and which has the added benefit of increasing the moment arm of the fins); to move the control surfaces farther forward to allow the fin wakes to diffuse out more before they encounter the blades, thereby decreasing the unsteady blade inflow pitch variation (the forward placement of the control surfaces is usually constrained by vehicle diameter limitations); and selecting rotor blade numbers which result in unsteady thrust forces but no unsteady side forces. This last alternative is based on the fact that at low frequencies the far-field propulsor radiated noise is largely due to reradiation by the vehicle hull of propulsor excitations and the assumption that it is easier to isolate the unsteady thrust forces from the hull than it is to isolate the unsteady side forces.

The results presented herein only address the frequencies associated with the interaction of uniformly spaced blade rows. Two extensions of this work immediately suggest themselves. The first is the issue of the interaction of a blade row of uniformly spaced blades with an inflow field whose harmonic character is more complicated than having a periodicity of $2\pi/M$. This issue is addressed in a report by Uhlman[4] wherein the general expressions for the blade response force and moment harmonics are derived. The second issue is that of the frequencies associated with blade rows with staggered or nonuniform blade to blade spacings. Such configurations are of great interest in the reduction of tonal forces, but have not yet been given extensive treatment. Discussion of such arrangements will have to await further work.

CONCLUSIONS

Following Strasberg and Breslin,[1] a general investigation of the frequencies at which the unsteady forces and moments will occur for multiple blade row propulsors has been presented. The results of this investigation have then been applied to propulsors with two blade rows, including not only counter-rotating blade rows but also preswirl and postswirl single rotation propulsors as well. The resulting frequencies have been presented for large numbers of blades in all of the above configurations. These results show that the configuration, as well as the blade numbers, is critically important in the determination of tonal frequencies to be expected.

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APPENDIX

EXTENDED TABLES OF PROPULSOR INTERACTION FREQUENCIES

Table A-1. Interaction Frequencies of Two Blade Rows Counterrotating at the Same Rate

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	2/4	3/5	5/7	7/9	9/11	11/13	13/15	15/17	17/19	19/21	21/23	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39
2	4/8	4/8	5/7	---	9/11	---	13/15	---	17/19	---	21/23	---	25/27	---	29/31	---	33/35	---	37/39
3	6/12	12/24	6/12	7/17	11/19	---	13/29	17/31	---	19/41	23/43	---	25/53	29/55	---	31/65	35/67	---	37/77
4	8/16	8/16	24/48	8/16	9/31	---	15/41	---	17/55	---	23/65	---	25/79	---	31/89	---	33/103	---	39/113
5	10/20	20/40	30/60	40/80	10/20	11/49	29/41	31/49	19/71	---	21/89	49/71	51/79	29/111	---	31/129	69/101	71/109	39/151
6	12/24	12/24	12/24	24/48	60/120	12/24	13/71	---	---	---	23/109	---	25/131	---	---	---	35/169	---	37/191
7	14/28	28/56	42/84	56/112	70/140	84/168	14/28	15/97	55/71	41/99	43/111	71/97	27/155	---	29/181	97/127	69/169	71/181	113153
8	16/32	16/32	48/96	16/32	80/160	48/96	112/224	16/32	17/127	---	65/111	---	79/129	---	31/209	---	33/239	---	113/191
9	18/36	36/72	18/36	72/144	90/180	36/72	126/252	144/288	18/36	19/161	89/109	---	53/181	55/197	---	127/161	35/271	---	37/305
10	20/40	20/40	60/120	40/80	20/40	60/120	140/280	80/160	180/360	20/40	21/199	---	79/181	---	---	---	101/239	---	39/341
11	22/44	44/88	66/132	88/176	110/220	132/264	154/308	176/352	198/396	220/440	22/44	23/241	131/155	111/197	89/241	65/287	67/307	109/287	153/265
12	24/48	24/48	24/48	24/48	120/240	24/48	168/336	48/96	72/144	120/240	264/528	24/48	25/287	---	---	---	169/239	---	191/265
13	26/52	52/104	78/156	104/208	130/260	156/312	182/364	208/416	234/468	260/520	286/572	312/624	26/52	27/337	181/209	129/287	103/339	181/287	77/417
14	28/56	28/56	84/168	56/112	140/280	84/168	28/56	112/224	252/504	140/280	308/616	168/336	364/728	28/56	29/391	---	169/307	---	113/419
15	30/60	60/120	30/60	120/240	30/60	60/120	210/420	240/480	90/180	60/120	330/660	120/240	390/780	420/840	30/60	31/449	239/271	---	151/419
16	32/64	32/64	96/192	32/64	160/320	96/192	224/448	32/64	288/576	160/320	352/704	96/192	416/832	224/448	480/960	32/64	33/511	---	191/417
17	34/68	68/136	102/204	136/272	170/340	204/408	238/476	272/544	306/612	340/680	374/748	408/816	442/884	476/952	510/1020	544/1088	34/68	35/577	305/341
18	36/72	36/72	36/72	72/144	180/360	36/72	252/504	144/288	36/72	180/360	396/792	72/144	468/936	252/504	180/360	288/576	612/1224	36/72	37/647
19	38/76	76/152	114/228	152/304	190/380	228/456	266/532	304/608	342/684	380/760	418/836	456/912	494/988	532/1064	570/1140	608/1216	646/1292	684/1368	38/76

Table A-2. Interaction Frequencies of a Rotating Aft Blade Row with a Nonrotating Forward Blade Row ($M \geq N$)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1/2	1/3	2/4	3/5	4/6	5/7	6/8	7/9	8/10	9/11	10/12	11/13	12/14	13/15	14/16	15/17	16/18	17/19	18/20
2	2/4	2/4	2/4	---	4/6	---	6/8	---	8/10	---	10/12	---	12/14	---	14/16	---	16/18	---	18/20
3	3/6	6/12	3/6	3/9	6/9	---	6/15	9/15	---	9/21	12/21	---	12/27	15/27	---	15/33	18/33	---	18/39
4	4/8	4/8	12/24	4/8	4/16	---	8/20	---	8/28	---	12/32	---	12/40	---	16/44	---	16/52	---	20/56
5	5/10	10/20	15/30	20/40	5/10	5/25	15/20	15/25	10/35	---	10/45	25/35	25/40	15/55	---	15/65	35/50	35/55	20/75
6	6/12	6/12	6/12	12/24	30/60	6/12	6/36	---	---	---	12/54	---	12/66	---	---	---	18/84	---	18/96
7	7/14	14/28	21/42	28/56	35/70	42/84	7/14	7/49	28/35	21/49	21/56	35/49	14/77	---	14/91	49/63	35/84	35/91	56/77
8	8/16	8/16	24/48	8/16	40/80	24/48	56/112	8/16	8/64	---	32/56	---	40/64	---	16/104	---	16/120	---	56/96
9	9/18	18/36	9/18	36/72	45/90	18/36	63/126	72/144	9/18	9/81	45/54	---	27/90	27/99	---	63/81	18/135	---	18/153
10	10/20	10/20	30/60	20/40	10/20	30/60	70/140	40/80	90/180	10/20	10/100	---	40/90	---	---	---	50/120	---	20/170
11	11/22	22/44	33/66	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	11/121	66/77	55/99	44/121	33/143	33/154	55/143	77/132
12	12/24	12/24	12/24	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	12/144	---	---	---	84/120	---	96/132
13	13/26	26/52	39/78	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26	13/169	91/104	65/143	52/169	91/143	39/208
14	14/28	14/28	42/84	28/56	70/140	42/84	14/28	56/112	126/252	70/140	154/308	84/168	182/364	14/28	14/196	---	84/154	---	56/210
15	15/30	30/60	15/30	60/120	15/30	30/60	105/210	120/240	45/90	30/60	165/330	60/120	195/390	210/420	14/30	15/225	120/135	---	75/210
16	16/32	16/32	48/96	16/32	80/160	48/96	112/224	16/32	144/288	80/160	176/352	48/96	208/416	112/224	240/480	16/32	16/256	---	96/208
17	17/34	34/68	51/102	68/136	85/170	102/204	119/238	136/272	153/306	170/340	187/374	204/408	221/442	238/476	255/510	272/544	17/34	17/289	153/170
18	18/36	18/36	18/36	36/72	90/180	18/36	126/252	72/144	18/36	90/180	198/396	36/72	234/468	126/252	90/180	144/288	306/612	18/36	18/324
19	19/38	38/76	57/114	76/152	95/190	114/228	133/266	152/304	171/342	190/380	209/418	228/456	247/494	266/532	285/570	304/608	323/646	342/684	19/38

Table A-3. Interaction Frequencies of a Rotating Aft Blade Row with a Nonrotating Forward Blade Row ($M \le N$)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1/2	2/4	3/6	4/8	5/10	6/12	7/14	8/16	9/18	10/20	11/22	12/24	13/26	14/28	15/45	16/32	17/34	18/36	19/38
2	2/4	2/4	3/9	---	5/15	---	7/21	---	9/27	---	11/33	---	13/39	---	15/45	---	17/51	---	19/57
3	3/6	6/12	3/6	4/8	5/10	---	7/14	8/16	---	10/20	11/22	---	13/26	14/28	---	16/32	17/34	---	19/38
4	4/8	4/8	12/24	4/8	5/15	---	7/21	---	9/27	---	11/33	---	13/39	---	15/45	---	17/51	---	19/57
5	5/10	10/20	15/30	20/40	5/10	6/24	14/21	16/24	9/36	---	11/44	24/36	26/39	14/56	---	16/64	34/51	36/54	19/76
6	6/12	6/12	6/12	12/24	30/60	6/12	7/35	---	---	---	11/55	---	13/65	---	---	---	17/85	---	19/95
7	7/14	14/28	21/42	28/56	35/70	42/84	7/14	8/48	27/36	20/50	22/55	36/48	13/78	---	15/90	48/64	34/85	36/90	57/76
8	8/16	8/16	24/48	8/16	40/80	24/48	56/112	8/16	9/63	---	33/55	---	39/65	---	15/105	---	17/119	---	57/95
9	9/18	18/36	9/18	36/72	45/90	18/36	63/126	72/144	9/18	10/80	44/55	---	26/91	28/98	---	64/80	17/136	---	19/152
10	10/20	10/20	30/60	20/40	10/20	30/60	70/140	40/80	90/180	10/20	11/99	---	39/91	---	---	---	51/119	---	19/171
11	11/22	22/44	33/66	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	12/120	65/78	56/98	45/120	32/144	34/153	54/144	76/133
12	12/24	12/24	12/24	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	13/143	---	---	---	85/119	---	95/133
13	13/26	26/52	39/78	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26	14/168	90/105	64/144	51/170	90/144	38/209
14	14/28	14/28	42/84	28/56	70/140	42/84	14/28	56/112	126/252	70/140	154/308	84/168	182/364	14/28	15/195	---	85/153	---	57/209
15	15/30	30/60	15/30	60/120	15/30	30/60	105/210	120/240	45/90	30/60	165/330	60/120	195/390	210/420	15/30	16/224	119/136	---	76/209
16	16/32	16/32	48/96	16/32	80/160	48/96	112/224	16/32	144/288	80/160	176/352	48/96	208/416	112/224	240/480	16/32	17/255	---	95/209
17	17/34	34/68	51/102	68/136	85/170	102/204	119/238	136/272	153/306	170/340	187/374	204/408	221/442	238/476	255/510	272/544	17/34	18/288	152/171
18	18/36	18/36	18/36	36/72	90/180	18/36	126/252	72/144	18/36	90/180	198/396	36/72	234/468	126/252	90/180	144/288	306/612	18/36	19/323
19	19/38	38/76	57/114	76/152	95/190	114/228	133/266	152/304	171/342	190/380	209/418	228/456	247/494	266/532	285/570	304/608	323/646	342/684	19/38

Table A-4. Interaction Frequencies of a Rotating Forward Blade Row with a Nonrotating Aft Blade Row ($M \geq N$)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1/2	2/4	3/6	4/8	5/10	6/12	7/14	8/16	9/18	10/20	11/22	12/24	13/26	14/28	15/45	16/32	17/34	18/36	19/38
2	2/4	2/4	3/9	---	5/15	---	7/21	---	9/27	---	11/33	---	13/39	---	15/45	---	17/51	---	19/57
3	3/6	6/12	3/6	4/8	5/10	---	7/14	8/16	---	10/20	11/22	---	13/26	14/28	---	16/32	17/34	---	19/38
4	4/8	4/8	12/24	4/8	5/15	---	7/21	---	9/27	---	11/33	---	13/39	---	15/45	---	17/51	---	19/57
5	5/10	10/20	15/30	20/40	5/10	6/24	14/21	16/24	9/36	---	11/44	24/36	26/39	14/56	---	16/64	34/51	36/54	19/76
6	6/12	6/12	6/12	12/24	30/60	6/12	7/35	---	---	---	11/55	---	13/65	---	---	---	17/85	---	19/95
7	7/14	14/28	21/42	28/56	35/70	42/84	7/14	8/48	27/36	20/50	22/55	36/48	13/78	---	15/90	48/64	34/85	36/90	57/76
8	8/16	8/16	24/48	8/16	40/80	24/48	56/112	8/16	9/63	---	33/55	---	39/65	---	15/105	---	17/119	---	57/95
9	9/18	18/36	9/18	36/72	45/90	18/36	63/126	72/144	9/18	10/80	44/55	---	26/91	28/98	---	64/80	17/136	---	19/152
10	10/20	10/20	30/60	20/40	10/20	30/60	70/140	40/80	90/180	10/20	11/99	---	39/91	---	---	---	51/119	---	19/171
11	11/22	22/44	33/66	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	12/120	65/78	56/98	45/120	32/144	34/153	54/144	76/133
12	12/24	12/24	12/24	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	13/143	---	---	---	85/119	---	95/133
13	13/26	26/52	39/78	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26	14/168	90/105	64/144	51/170	90/144	38/209
14	14/28	14/28	42/84	28/56	70/140	42/84	14/28	56/112	126/252	70/140	154/308	84/168	182/364	14/28	15/195	---	85/153	---	57/209
15	15/30	30/60	15/30	60/120	15/30	30/60	105/210	120/240	45/90	30/60	165/330	60/120	195/390	210/420	15/30	16/224	119/136	---	76/209
16	16/32	16/32	48/96	16/32	80/160	48/96	112/224	16/32	144/288	80/160	176/352	48/96	208/416	112/224	240/480	16/32	17/255	---	95/209
17	17/34	34/68	51/102	68/136	85/170	102/204	119/238	136/272	153/306	170/340	187/374	204/408	221/442	238/476	255/510	272/544	17/34	18/288	152/171
18	18/36	18/36	18/36	36/72	90/180	18/36	126/252	72/144	18/36	90/180	198/396	36/72	234/468	126/252	90/180	144/288	306/612	18/36	19/323
19	19/38	38/76	57/114	76/152	95/190	114/228	133/266	152/304	171/342	190/380	209/418	228/456	247/494	266/532	285/570	304/608	323/646	342/684	19/38

Table A-5. Interaction Frequencies of a Rotating Forward Blade Row with a Nonrotating Aft Blade Row (M ≤N)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1/2	1/3	2/4	3/5	4/6	5/7	6/8	7/9	8/10	9/11	10/12	11/13	12/14	13/15	14/16	15/17	16/18	17/19	18/20
2	2/4	2/4	2/4	---	4/6	---	6/8	---	8/10	---	10/12	---	12/14	---	14/16	---	16/18	---	18/20
3	3/6	6/12	3/6	3/9	6/9	---	6/15	9/15	---	9/21	12/21	---	12/27	15/27	---	15/33	18/33	---	18/39
4	4/8	4/8	12/24	4/8	4/16	---	8/20	---	8/28	---	12/32	---	12/40	---	16/44	---	16/52	---	20/56
5	5/10	10/20	15/30	20/40	5/10	5/25	15/20	15/25	10/35	---	10/45	25/35	25/40	15/55	---	15/65	35/50	35/55	20/75
6	6/12	6/12	6/12	12/24	30/60	6/12	6/36	---	---	---	12/54	---	12/66	---	---	---	18/84	---	18/96
7	7/14	14/28	21/42	28/56	35/70	42/84	7/14	7/49	28/35	21/49	21/56	35/49	14/77	---	14/91	49/63	35/84	35/91	56/77
8	8/16	8/16	24/48	8/16	40/80	24/48	56/112	8/16	8/64	---	32/56	---	40/64	---	16/104	---	16/120	---	56/96
9	9/18	18/36	9/18	36/72	45/90	18/36	63/126	72/144	9/18	9/81	45/54	---	27/90	27/99	---	63/81	18/135	---	18/153
10	10/20	10/20	30/60	20/40	10/20	30/60	70/140	40/80	90/180	10/20	10/100	---	40/90	---	---	---	50/120	---	20/170
11	11/22	22/44	33/66	44/88	55/110	66/132	77/154	88/176	99/198	110/220	11/22	11/121	66/77	55/99	44/121	33/143	33/154	55/143	77/132
12	12/24	12/24	12/24	12/24	60/120	12/24	84/168	24/48	36/72	60/120	132/264	12/24	12/144	---	---	---	84/120	---	96/132
13	13/26	26/52	39/78	52/104	65/130	78/156	91/182	104/208	117/234	130/260	143/286	156/312	13/26	13/169	91/104	65/143	52/169	91/143	39/208
14	14/28	14/28	42/84	28/56	70/140	42/84	14/28	56/112	126/252	70/140	154/308	84/168	182/364	14/28	14/196	---	84/154	---	56/210
15	15/30	30/60	15/30	60/120	15/30	30/60	105/210	120/240	45/90	30/60	165/330	60/120	195/390	210/420	14/30	15/225	120/135	---	75/210
16	16/32	16/32	48/96	16/32	80/160	48/96	112/224	16/32	144/288	80/160	176/352	48/96	208/416	112/224	240/480	16/32	16/256	---	96/208
17	17/34	34/68	51/102	68/136	85/170	102/204	119/238	136/272	153/306	170/340	187/374	204/408	221/442	238/476	255/510	272/544	17/34	17/289	153/170
18	18/36	18/36	18/36	36/72	90/180	18/36	126/252	72/144	18/36	90/180	198/396	36/72	234/468	126/252	90/180	144/288	306/612	18/36	18/324
19	19/38	38/76	57/114	76/152	95/190	114/228	133/266	152/304	171/342	190/380	209/418	228/456	247/494	266/532	285/570	304/608	323/646	342/684	19/38

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