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Calculation of the sound generated by the head-on collision of two vortex rings

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Abstract

The dynamics of the head-on collision of two identical rings vortices is computed using an axisymmetric solution of the vorticity equation on a Lagrangian mesh. The vorticity and velocity distributions determined in this manner are then used to compute the radiated sound produced by this collision. The predicted sound is compared to measurements and the agreement is seen to be excellent.

Keywords: Vorticity; Vortex; Ring vortex; Lagrangian; Sound; Vortex sound

1. Introduction

The present work grew out of an effort to predict the sound radiated by aircraft trailing vortices in order to determine an acoustic signature by which they might be tracked. Detection and location of these vortices is an important factor in the effort to decrease the time between aircraft landings and thereby increase airport capacity. We elected to employ a Lagrangian formulation of the vorticity equation in order to significantly reduce the effects of numerical diffusion and allow the vortices to be followed computationally for sufficient time that the physics governing the acoustic radiation could be resolved. In order to test these ideas, we developed an axisymmetric version of the approach to eliminate other possible numerical issues and allow application to the problem studied by Kambe and Minota [1].

2. Formulation

The Lagrangian form of the axisymmetric displacement and vorticity equations which are to be solved are

$$\frac{\mathrm{d}z}{\mathrm{d}t} = w, \qquad \frac{\mathrm{d}r}{\mathrm{d}t} = q, \qquad \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\omega q}{r} \tag{1}$$

where z and r are the axial and radial coordinates, respectively, w and q are the axial and radial velocity components, respectively, and ω is the magnitude of the circumferential component of vorticity. All quantities are made dimensionless with respect to the ambient fluid density, ρ_0, the initial vortex core radius, σ and the circulation about the vortex core, Γ. The solution proceeds by providing an initial triangular mesh and vorticity distribution for the region of non-zero vorticity. The above equations are then solved using an embedded, fourth-order, adaptive time-step Runge–Kutta scheme. The nodal points of the elements are advected with the local velocity. Remeshing is required to avoid severely distorted meshes. The velocities are computed using the Biot–Savart law with the appropriate axisymmetric kernel functions (see Appendix A). Special numerical integration schemes are employed to evaluate the resulting singular and near-singular integrals. The second vortex is computed as an image across the plane of symmetry.

Once the velocity and vorticity distributions have been determined the radiated sound is computed using (see [2])

$$p' \cong \frac{\rho_0}{4\pi c_0^2} \frac{x_i x_j}{R^3} \frac{\partial^2}{\partial t^2} \iiint_V \left[\xi_i \left(\varepsilon_{jkl} u_k \omega_l\right)\right] \mathrm{d}V + \frac{\rho_0}{4\pi c_0^2} \frac{1}{R} \frac{\partial^2}{\partial t^2} \iiint_V \left[\tfrac{1}{2} u_i u_i\right] \mathrm{d}V \tag{2}$$

where $R = \sqrt{x_j x_j}$, c_0 is the small disturbance sound speed and it has been assumed that the source region is compact and that the observation point is in the far-field. For the inviscid calculation considered here the second term in Eq. (2) should be zero since the kinetic energy of the flow should be conserved. This turns out to be the case to good accuracy. The integrals required in Eq. (2) are computed from the solution of Eq. (1). The required time derivatives

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are then evaluated using a Savitsky–Golay filter (see [3]) and the radiated sound computed at the desired observation point.

3. Results

Measurements of the sound produced by the head-on collision of two identical ring vortices have been made by Kambe and Minota [1]. Calculations were performed to reproduce the conditions of their experiment in order to permit comparison. To this end we computed the interaction of rings vortices with $R/\sigma = 2$ where R is the ring radius and σ is the core radius. The steady translation solution of Norbury [4] was used as the initial condition.

Fig. 1 shows a cut-away view of the vortex ring collision process demonstrating the rapid increase in ring radius which occurs and drives the increase in the vorticity due to stretching.

Figs. 2–4 present the results of the flow calculation showing the evolution with time of the vorticity, the axial and radial velocity components and the pressure. We note that the vorticity remains proportional to radius as required by the vorticity equation for a Norbury initialization. The velocity components are, in contrast, distinctly non-uniform with very high radial velocities developing near the plane of symmetry as the two vortices approach one another.

Fig. 5 presents a comparison of the computed sound pressure at an observation point on the plane of symmetry

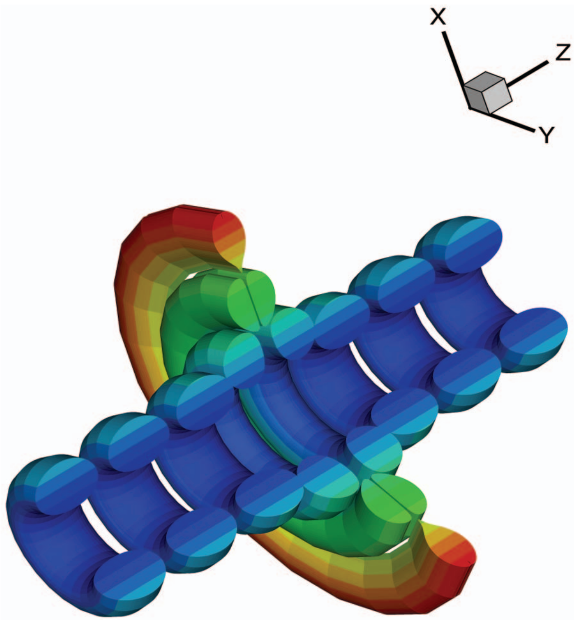

Fig. 1. Cutaway figure of head-on collision of two axisymmetric ring vortices. Colors denote magnitude of circumferential vorticity.

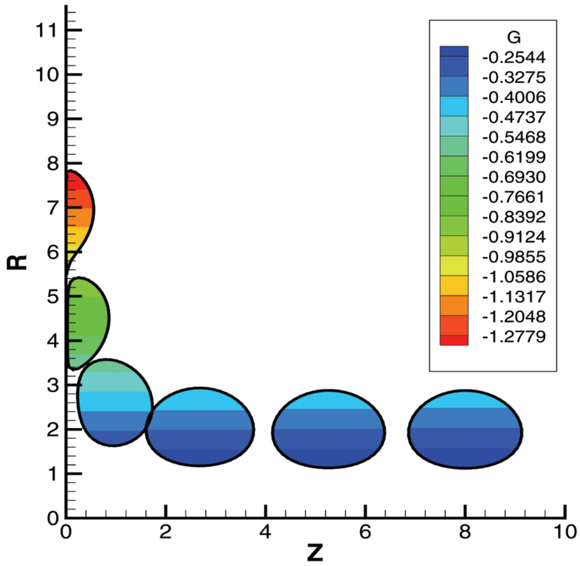

Fig. 2. Cross-sectional view of head-on collision of two axisymmetric ring vortices. Colors denote magnitude of circumferential vorticity.

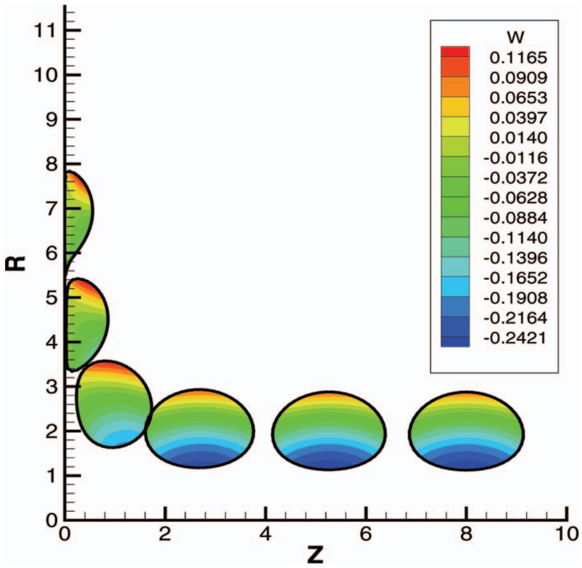

Fig. 3. Cross-sectional view of head-on collision of two axisymmetric ring vortices. Colors denote magnitude of axial component of velocity.

at a dimensionless radius of 184 and a dimensionless sound speed of $c_0\sigma/\Gamma = 0.6007$ (note that this corresponds to a Mach number based on initial ring propagation speed of roughly 0.17). The dimensionless time and pressure are defined by $\Gamma t/\sigma^2$ and $p'\sigma^2/\rho_0\Gamma^2$, respectively. The calculation becomes difficult to continue after a dimensionless time of about 105 due to the near-singular behavior of the

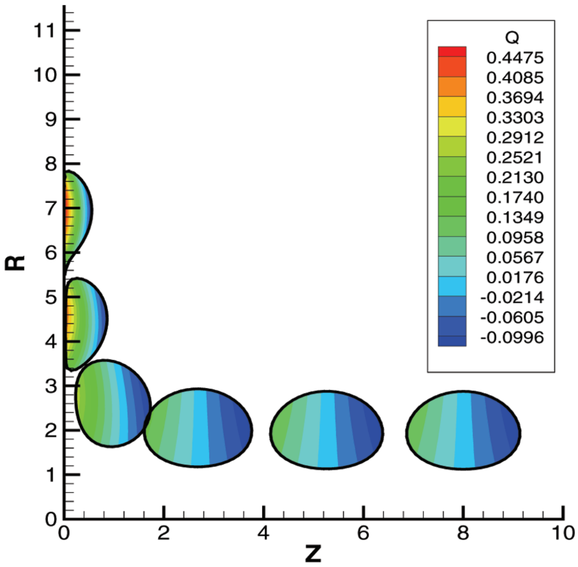

Fig. 4. Cross-sectional view of head-on collision of two axisymmetric ring vortices. Colors denote magnitude of radial component of velocity.

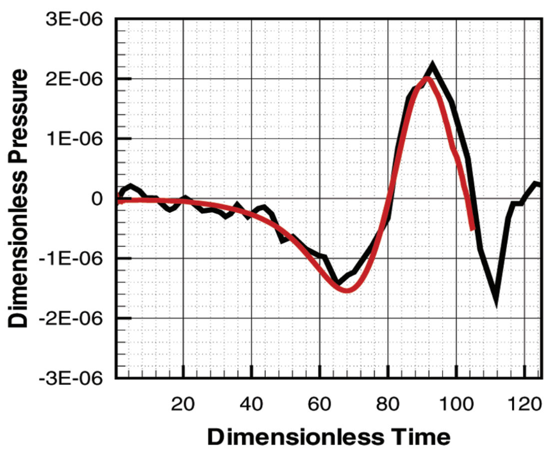

Fig. 5. Comparison of levels of dimensionless sound pressure versus dimensionless time as measured and predicted at the observation point. The black line represents the measured data of Kambe and Minota [1] and the red line represents the levels predicted by the present method.

integrals as the vortices approach the plane of symmetry. The agreement between the predicted and the measured sound pressures is seen to be excellent through the time computed. It should be noted that examination of other experiments (see e.g. [5]) suggest that the assumption of axisymmetric behavior may also begin to fail at about this time leading one to believe that further extension of the solution might no longer match the data in any event

4. Conclusions

The present work demonstrates the ability of a Lagrangian, vorticity-based method to compute the flow field of a ring vortex with sufficient accuracy to permit the calculation of the radiated sound. Although others have computed the radiated sound due to the head-on collision of two ring vortices (see e.g. [7]) the present work represents an advance due to the ability of the current method to handle arbitrary vorticity distributions and its ability to be extended to three dimensional vortex structures (see e.g. [8–13]).

Acknowledgements

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Appendix A. Axisymmetric velocity integrals

The velocities employed in the calculation are obtained from

$$
\begin{aligned}
q &= -\tfrac{1}{4\pi}\iint_S \left[(z-\zeta)\, J_3^1(A,B)\right]\omega(\rho,\zeta)\,\rho\,\mathrm{d}\rho\,\mathrm{d}\zeta \\
w &= +\tfrac{1}{4\pi}\iint_S \left[r J_3^1(A,B) - \rho J_3^0(A,B)\right]\omega(\rho,\zeta)\,\rho\,\mathrm{d}\rho\,\mathrm{d}\zeta
\end{aligned} \tag{A1}
$$

where

$$J_n^m(A,B) = \int_{-\pi}^{+\pi} \frac{\cos^m(\varphi)}{[A - B\cos(\varphi)]^{n/2}}\,\mathrm{d}\varphi \tag{A2}$$

and

$$A = r^2 + \rho^2 + (z-\zeta)^2 \qquad B = 2r\rho \tag{A3}$$

Using Eq. (A4) we can derive the recursion relation

$$J_n^m(A,B) = \frac{1}{B}\left[A J_n^{m-1}(A,B) - J_{n-2}^{m-1}(A,B)\right] \tag{A4}$$

so that we obtain

$$
\begin{aligned}
J_1^0(A,B) &= \frac{4}{\sqrt{A+B}} K(k) \\
J_3^0(A,B) &= \frac{4}{(A-B)\sqrt{A+B}} E(k) \\
J_3^1(A,B) &= \frac{1}{B}\left[A J_3^0(A,B) - J_1^0(A,B)\right]
\end{aligned} \tag{A5}
$$

where $k^2 = 2B/(A+B)$ and $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively.

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